

The radius and mass of the subgiant star β Hyi from interferometry and asteroseismology

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1 February 2008

ABSTRACT

We have used the Sydney University Stellar Interferometer (SUSI) to measure the angular diameter of β Hydri. This star is a nearby G2 subgiant whose mean density was recently measured with high precision using asteroseismology. We determine the radius and effective temperature of the star to be $1.814 \pm 0.017 R_{\odot}$ (0.9%) and 5872 ± 44 K (0.7%) respectively. By combining this value with the mean density, as estimated from asteroseismology, we make a direct estimate of the stellar mass. We find a value of $1.07 \pm 0.03 M_{\odot}$ (2.8%), which agrees with published estimates based on fitting in the H-R diagram, but has much higher precision. These results place valuable constraints on theoretical models of β Hyi and its oscillation frequencies.

Key words: stars: individual: β Hyi – stars: fundamental parameters – techniques: interferometric

1 INTRODUCTION

The combination of interferometry and asteroseismology is a powerful way of constraining the parameters of stars (Kervella et al. 2003; Pijpers et al. 2003; Kervella et al. 2004; Thévenin et al. 2005; van Belle, Ciardi, & Boden 2007; Creevey et al. 2007). Interferometry provides an angular diameter which, combined with the parallax, gives a direct measurement of the stellar radius. Asteroseismology, on the other hand, uses the oscillation frequencies of the star to infer details about its internal structure (e.g., Brown & Gilliland 1994). In particular, $\Delta\nu$, the so-called large frequency separation between consecutive radial overtones, gives a good estimate of the mean stellar density. The combination of the radius and $\Delta\nu$ can therefore provide a direct measurement of the mass of a star.

The star β Hyi (HR 98, HD 2151, HIP 2021) is a southern evolved subgiant (spectral type G2 IV) with a mass slightly greater than the Sun and an age of about 6.5–7.0 Gy (Dravins et al. 1998; Fernandes & Monteiro 2003). This star is an excellent target for asteroseismology, being the closest G-type subgiant. Its distance, thanks to *Hipparcos*, is known to 0.36% accuracy. It is bright ($V = 2.8$) and has a low $v \sin i$, which allows extremely good Doppler precision. Solar-like oscillations in β Hyi were recently measured by Bedding et al. (2007), who used the large frequency separation to infer the mean stellar density to an accuracy of 0.6%.

In this paper we report the first measurement of the angular diameter of β Hyi. We use the *Hipparcos* parallax to infer the radius, which we combine with the mean density, determined from asteroseismology, to estimate the mass of the star. The effective temper-

ature and surface gravity are also constrained using our radius and complementary data from the literature.

2 OBSERVATIONS AND DATA REDUCTION

The Sydney University Stellar Interferometer (SUSI, Davis et al. 1999) was used to measure the squared visibility (i.e. normalised squared modulus of the complex visibility) or V^2 on a total of 7 nights. The red-table beam-combination system was employed with a filter of centre wavelength and full-width half-maximum 700 nm and 80 nm respectively. This system is to be described in greater detail by Davis et al. (in preparation) and an outline is given by Tuthill et al. (2004).

The interference produced by a pupil-plane beam-combiner was modulated by repeatedly scanning the optical delay about the white light fringe position. Two avalanche photo-diodes detect the two outputs of the combining beamsplitter. An observation consisted of a set of 1000 scans, each traversing $140 \mu\text{m}$ in optical delay, sampled in 1024 steps of 0.2 ms duration.

The post-processing software mitigates the effect of scintillation in each observation by differencing the two recorded signals. The squared visibility was estimated after bias subtraction and a nonlinear function was applied to partially correct for residual seeing effects (Ireland 2006).

Target observations were interleaved with calibration measurements of nearby stars chosen to provide a known or essentially unresolved signal. The angular diameters (and associated error) of these calibrator stars were estimated from an intrinsic colour

Table 1. Adopted parameters of calibrators used during observations.

HR	Name	Spectral Type	V	UD Diameter (mas)	separation from β Hyi
0705	δ Hyi	A3V	4.08	0.50 ± 0.03	$11^\circ 85$
0806	ϵ Hyi	B9V	4.10	0.39 ± 0.03	$13^\circ 05$
7590	ϵ Pav	A0V	3.96	0.46 ± 0.05	$16^\circ 53$

interpolation (and spread in data) of measurements made with the Narrabri Stellar Intensity Interferometer (Hanbury Brown et al. 1974), including corrections for the effects of limb-darkening. Using the adopted stellar parameters of the calibrator stars given in Table 1, the system response or transfer function was quantified. The weighted mean of the bracketing transfer functions was then used to scale the observed target squared visibility appropriately to produce measurements of V^2 . This procedure resulted in a total of 35 estimations of V^2 and a summary of each night is given in Table 2.

It should be noted that the uncertainties in the calibrator angular diameters, when compared to the measurement uncertainty, have a negligible effect on the final V^2 but for consistency, were included in the uncertainty calculation.

3 ANGULAR DIAMETER

The brightness distribution of a star can be modeled, in the simplest case, to be a disc of uniform irradiance with angular diameter θ_{UD} . The theoretical response of a two aperture interferometer to such a model is given by

$$|V|^2 = \left| \frac{2J_1(\pi|b|\theta_{UD}/\lambda)}{\pi|b|\theta_{UD}/\lambda} \right|^2, \quad (1)$$

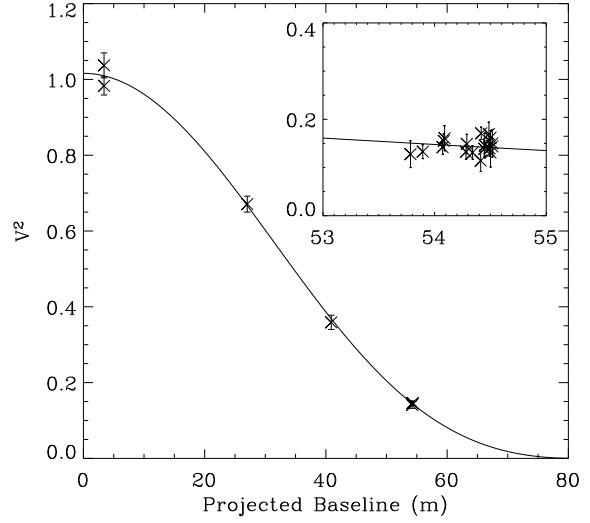
where J_1 is a first order Bessel function, b is the baseline projected onto the plane of the sky and λ is the observing wavelength. For stars that have a compact atmosphere only small corrections are needed to account for monochromatic limb-darkening – these corrections can be found in Davis, Tango & Booth (2000).

An additional parameter, A , is included in the uniform disc model to account for instrumental effects arising from the differing spectral types of β Hyi and the calibrators. Equation (1) becomes

$$|V|^2 = \left| A \frac{2J_1(\pi|b|\theta/\lambda)}{\pi|b|\theta/\lambda} \right|^2. \quad (2)$$

The estimation of θ_{UD} and A was completed using χ^2 minimisation with an implementation of the Levenberg-Marquardt method to fit equation (2) to all measures of V^2 . The formal uncertainties derived from the diagonal elements of the covariance matrix (which are calculated as part of the χ^2 minimisation) were verified by *Monte Carlo* simulations – the visibility measurement errors may not strictly conform to a normal distribution and equation (2) is non-linear. These simulations subjected the model visibilities of each observation to a Monte Carlo realisation of the (assumed) normal measurement error distribution to produce synthetic data sets. Estimates of the model parameters are then found using the synthetic datasets, thus building a distribution of each parameter.

The reduced χ^2 of the fit was 1.29 implying that the measurement uncertainties were underestimated. We have therefore scaled the uncertainties in the fit parameters by $\sqrt{1.29}$ to obtain a reduced χ^2 of unity. Final values of θ_{UD} and A (with the associated 1σ

**Figure 1.** Nightly mean V^2 measures with the fitted uniform disc overlaid. Inset: all V^2 measures at the (nominal) 80 m SUSI baseline.

parameter uncertainty) are 2.156 ± 0.017 mas and 1.008 ± 0.007 respectively. The data, with the fitted uniform disc model overlaid, are shown in Figure 1. The fitted value of A is close to unity indicating that the effect of the differing spectral types of the stars is very small and can be neglected. Furthermore, the resolution of SUSI during observations was such that the calibrator stars were essentially unresolved.

Analysis of the effect of the wide observing band was completed using a G2 IV flux distribution (Davis et al. in preparation). Wide bandwidth effects are only significant when the interferometer's coherent field-of-view is smaller than the angular extent of source (Tango & Davis 2002). The coherent field-of-view during observations was found to be greater than 7 mas hence *bandwidth smearing* can be considered negligible. However, the interferometer's *effective* wavelength when observing a G2 IV star is approximately 696.6 ± 2.0 nm (Davis et al. in preparation). Subsequently, the final fit to equation (2) was completed with the new effective wavelength. Using the work of Davis et al. (2000), the limb-darkening correction was determined to be 1.047 using the following parameters: $T_{\text{eff}} = 5872$ K, $\log g = 3.95$ and $[\text{Fe}/\text{H}] = -0.17$. These parameters (with the exception of $[\text{Fe}/\text{H}]$) are the result of an iterative procedure whereby initial values, obtained from SIMBAD, were refined by subsequent fundamental parameter determination (see Section 4). We note, however, that the change in T_{eff} and $\log g$ only resulted in a decrement of the last digit in the limb-darkening correction. The parameter $[\text{Fe}/\text{H}]$ is the mean of the values listed on SIMBAD and we estimate an uncertainty in the limb-darkening correction of 0.002. Therefore the limb-darkened diameter is $\theta_{LD} = 2.257 \pm 0.019$ mas and carries the caveat that the Kurucz models that Davis et al. (2000) based their work upon are accurate for this star.

4 DISCUSSION

Formulae to calculate the stellar luminosity (L), effective temperature (T_{eff}) and radius (R) from the observable quantities angular diameter (θ), bolometric flux (f) and parallax (π_p) are:

Table 2. Summary of observational data. The night of the observation is given in Columns 1 and 2 as a calendar date and a mean MJD. The nominal and mean projected baseline in units of metres is given in Columns 3 and 4 respectively. The mean squared visibility, standard deviation and number of observations during a night is given in the last three columns.

Date	MJD	Nominal Baseline	Mean Projected Baseline	Calibrators	\bar{V}^2	$\bar{\sigma}$	# V^2
2004 October 08	53286.56	60	40.87	ϵ Pav, δ Hyi, ϵ Hyi	0.359	0.019	2
2004 October 29	53307.48	5	3.40	ϵ Pav, δ Hyi	1.037	0.033	4
2005 September 20	53633.64	40	27.08	ϵ Pav, δ Hyi	0.671	0.021	6
2005 November 11	53685.45	80	54.40	ϵ Pav, δ Hyi, ϵ Hyi	0.145	0.005	6
2005 November 12	53686.42	80	54.23	ϵ Pav, δ Hyi, ϵ Hyi	0.141	0.009	5
2005 November 13	53687.47	80	54.29	ϵ Pav, δ Hyi, ϵ Hyi	0.146	0.006	7
2005 November 27	53701.43	5	3.40	ϵ Pav, δ Hyi, ϵ Hyi	0.983	0.024	5

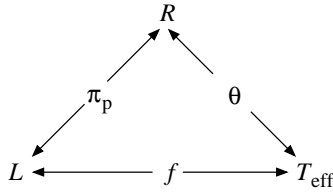


Figure 2. Relationships between the observable quantities (inner) and fundamental stellar parameters at the vertices.

$$L = 4\pi f \frac{C^2}{\pi_p^2}, \quad (3)$$

$$T_{\text{eff}} = \left[\frac{4f}{\sigma\theta^2} \right]^{1/4}, \quad (4)$$

$$R = 2\theta \frac{C}{\pi_p}, \quad (5)$$

where σ is the Stefan-Boltzmann constant and C is the conversion from parsecs to metres. The relationship between these quantities can be visualised as Fig. 2, where the inner quantities are the observables and the vertices are the fundamental stellar parameters.

Our measurement of the angular diameter of β Hyi, after correcting for limb darkening, has an accuracy of 0.8%. Combining this with the *Hipparcos* parallax (0.4%) allows us to determine the radius of the star with an accuracy of 0.9%, as given in Table 3. This radius determination locates the star in the H-R diagram with much greater accuracy than was previously possible (see Fig. 3) and will be extremely important for calculating theoretical models for β Hyi.

Our angular diameter measurement, in combination with the bolometric flux of Blackwell & Lynas-Gray (1998) (the associated uncertainty estimation has been taken from Di Benedetto 1998) yields an effective temperature of 5872 ± 44 K. This value, with an accuracy of 0.7%, is higher than those presented in Blackwell & Lynas-Gray (1998) and Di Benedetto (1998): 5710 ± 29 K and 5774 ± 52 K respectively. While all three temperature estimations use the bolometric flux of Blackwell & Lynas-Gray (1998), only our value uses a direct measurement of the angular diameter (and hence radius) of β Hyi. Furthermore, a recent calibration of the MK system for late A-, F- and early G-type stars by Gray et al. (2001) yields a value of 5850 K for a G2 IV star and the recent spectroscopic analysis by da Silva et al. (2006) produced an effective temperature of 5964 ± 70 K for β Hyi, both of which are consistent with our value.

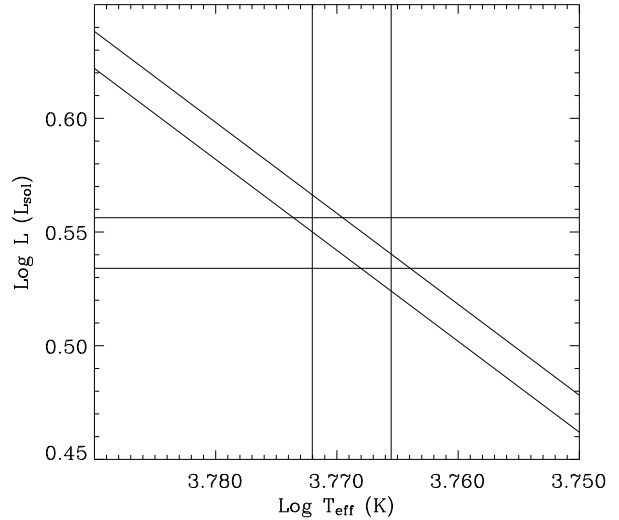


Figure 3. The 1-sigma constraints presented in this work on the effective temperature (vertical), luminosity (horizontal) and radius (diagonal), shown in an H-R diagram.

The luminosity of β Hyi, found from the *Hipparcos* parallax and bolometric flux, is $3.51 \pm 0.09 L_{\odot}$ (following Bahcall, Pinsonneault & Basu 2001, we adopt $L_{\odot} = 3.842 \times 10^{26}$ W with an uncertainty of 0.4%). The constraints these fundamental stellar values place on the location of β Hyi in the H-R diagram are shown in Fig 3.

Calculating theoretical models for β Hyi is beyond the scope of this paper. However, we are able to combine our radius measurement with the mean density, inferred from the observed value of the large frequency separation in the asteroseismic data, to determine the mass using:

$$M = \frac{4}{3} \pi \bar{\rho} R^3. \quad (6)$$

Taking the value for the mean density of β Hyi, determined to a precision of 0.6% by Bedding et al. (2007) using asteroseismology (see Table 3), we calculate the mass to be $1.07 \pm 0.03 M_{\odot}$. This is consistent with, but much more precise than, values in the literature that were estimated from modelling the position of β Hyi in the H-R diagram: $1.1 M_{\odot}$ (Dravins et al. 1998), $1.10^{+0.04}_{-0.07} M_{\odot}$ (Fernandes & Monteiro 2003), $1.17 \pm 0.05 M_{\odot}$ (da Silva et al.

Table 3. Physical parameters of β Hyi.

Parameter	Value	Uncertainty (%)	Source
θ_{LD} (mas)	2.257 ± 0.019	0.8	this work
π_p (mas)	133.78 ± 0.51	0.4	ESA97
f (10^{-9} W m^{-2})	2.019 ± 0.050	2.5	BLG98, DiB98
L (L_{\odot})	3.51 ± 0.09	2.6	this work
T_{eff} (K)	5872 ± 44	0.7	this work
R (R_{\odot})	1.814 ± 0.017	0.9	this work
$\bar{\rho}$ ($\bar{\rho}_{\odot}$)	0.1803 ± 0.0011	0.6	BKA07
M (M_{\odot})	1.07 ± 0.03	2.8	this work
$\log g$	3.952 ± 0.005	0.1	this work

ESA97: ESA (1997); BLG98: Blackwell & Lynas-Gray (1998);
DiB98: Di Benedetto (1998); BKA07: Bedding et al. (2007)

2006). Finally, we can combine our radius with the mean density to produce an estimate of the surface gravity,

$$g = \frac{4}{3} G \pi \bar{\rho} R, \quad (7)$$

where G is the universal constant of gravitation. The value we obtain for the surface gravity leads to $\log g = 3.952 \pm 0.005$, which has a precision of 0.1%.

5 CONCLUSION

We have presented the first angular diameter measurement of the G2 subgiant β Hyi. In combination with literature values for the bolometric flux and parallax, the angular diameter has constrained the stellar radius and effective temperature. The radius measurement, combined with the mean density determined from asteroseismology, allows the most accurate mass estimate of β Hyi to date. Indeed, this is perhaps the most precise mass determination of a solar-type star that is not in a binary system (apart from the Sun).

The constraints on L , T_{eff} , R , M and $\log g$ that we present will be invaluable in the future to critically test theoretical models of β Hyi and its oscillations. As stressed by Brown & Gilliland (1994), for example, oscillation frequencies are of most importance for testing evolution theories when the other fundamental stellar properties are well-constrained.

ACKNOWLEDGMENTS

This research has been jointly funded by The University of Sydney and the Australian Research Council as part of the Sydney University Stellar Interferometer (SUSI) project. JRN and APJ acknowledge the support provided by a University of Sydney Postgraduate Award. APJ and SMO were supported by a Denison Postgraduate Award. This research has made use of the SIMBAD database, operated at CDS, Strasbourg, France.

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